PERGAMON

International Journal of Heat and Mass Transfer 43 (2000) 2707–2710

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Opposite-current flows in gas–liquid boundary layers — II. Mass transfer kinetics

M. Doichinova, Chr. Boyadjiev*

Bulgarian Academy of Science, Institute of Chemical Engineering, Sofia 1113, Bl. 103, Bulgaria

Received 15 October 1999

Abstract

A theoretical analysis of mass transfer kinetics based on similarity variables method for gas–liquid counter-current flow has been done. The obtained numerical results for the mass transfer rate (Sherwood number) in case of a laminar boundary layer with flat phase boundary are compared with analogous results for co-current flow. The ratio between the mass transfer velocity and the dissipation energy in boundary layer is determined. The advantages of co-current flow because of lower energy losses than in case of counter-current one are shown. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

In the first part of the work, the velocity distribution in a laminar boundary layer of gas–liquid counter-current flow with flat phase boundary was determined [1]. The obtained numerical results are in a good agreement with results of the asymptotic theory and the experimental data. They allowed the determination of dissipation energy at the boundary layer and its comparison with the one in case of co-current flow. The subject of the present work is the influence on the mass transfer kinetics caused by the significant difference in the velocity profiles in laminar boundary layer in case of counter-current and co-current flows.

2. Mathematical model

The velocity of mass transfer in boundary layer is determined by the solution of the convective-diffusion equation. Boundary conditions at the phase boundary are introduced for the case of mass transfer between gas and liquid [3] to take into account the existence of thermodynamical equilibrium and continuity of mass flow. In this way, the mathematical model of mass transfer in gas–liquid systems with counter-current flow in a laminar boundary layer with flat phase boundary takes the following form:

$$u_i\frac{\partial c_i}{\partial x}+v_i\frac{\partial c_i}{\partial y}=D_i\frac{\partial^2 c_i}{\partial y^2},\quad i=1,2;$$

$$x=0,\quad y\geq 0,\quad c_1=c_2^{\infty};$$

$$x=l,\quad y\leq 0,\quad c_2=c_2^{\infty};$$

$$y\to\infty,\quad 0\leq x\leq l,\quad c_1=c_1^{\infty};$$

$$y\to-\infty,\quad 0\leq x\leq l,\quad c_2=c_2^{\infty};$$

$$y=0,\quad 0<x<l,\quad c_1=\chi c_2,\quad D_1\frac{\partial c_1}{\partial y}=D_2\frac{\partial c_2}{\partial y},\qquad (1)$$

---

* Corresponding author. Tel.: +359-270-4154; fax: +359-273-4936.

E-mail address: chboyadj@bas.bg (Chr. Boyadjiev).

PII: S0017-9310(99)00322-1

Nomenclature

c	concentration (kg mol/m^3)
D	diffusivity (m^2/s)
k	mass transfer coefficient (m/s)
u	velocity in x direction (m/s)
v	velocity in y direction (m/s)
x	coordinate (m)
y	coordinate (m)

Greek symbols

υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
χ	Henry constant

Subscripts

1	for gas
2	for liquid

Superscript

$*$	for co-current flow

where u_i and v_i $(i = 1, 2)$ are the velocity components in gaseous $(i = 1)$ and liquid phase $(i = 2)$ determined in Ref. [1], and c_i^∞ $(i = 1, 2)$ are the input concentrations of absorbed substance.

3. Method of solution

The solution of problem (1) should be made [3,4] after the introduction of similarity variables, used in Ref. [1]:

$$\eta_i = (-1)^{i+1} y \sqrt{\frac{u_i^\infty}{\upsilon_i l X_i}},$$

$$X_1 = \frac{x}{l}, \quad X_2 = \frac{l-x}{l}, \quad X_1 + X_2 = 1,$$

$$u_i = (-1)^{i+1} u_0^\infty f_i', \quad v_i = (-1)^{i+1} \frac{1}{2} \sqrt{\frac{\upsilon_i u_i^\infty}{l X_i}} \left(\eta_i f_i' - f_i\right),$$

$$f_i = f_i(\eta_i),$$

$$c_i = c_i^\infty - \chi^{1-i}\left(c_1^\infty - \chi c_2^\infty\right)\varphi_i, \quad \varphi_i = \varphi_i(\eta_i), \quad i = 1, 2. \tag{2}$$

Substitution of Eq. (2) into Eq. (1) leads to:

$$2f_i''' + f_i f_i'' = 0, \quad 2\varphi_i'' + Sc_i f_i \varphi_i' = 0, \quad i = 1, 2,$$

$$f_1(0) = 0, \quad f_1'(0) = a, \quad f_1''(0) = b,$$

$$f_2(0) = 0, \quad f_2'(0) = -\frac{a}{\theta_1}, \quad f_2''(0) = \bar{\theta}_2 b,$$

$$\varphi_1(0) + \varphi_2(0) = 1, \quad \bar{\theta}_3 \varphi_1'(0) = \varphi_2'(0), \varphi_i(\infty) = 0, \quad i = 1, 2, \tag{3}$$

where

$$Sc_i = \frac{\upsilon_i}{D_i} \quad (i = 1, 2), \quad \theta_1 = \frac{u_2^\infty}{u_1^\infty},$$

$$\theta_2 = \left(\frac{\rho_1 \mu_1}{\rho_2 \mu_2}\right)^{1/2} \left(\frac{u_1^\infty}{u_2^\infty}\right)^{3/2},$$

$$\bar{\theta}_2 = \theta_2 \sqrt{\frac{X_2}{X_1}}, \quad \theta_3 = \chi \frac{D_1}{D_2} \sqrt{\frac{u_1^\infty \upsilon_2}{u_2^\infty \upsilon_1}}, \quad \bar{\theta}_3 = \theta_3 \sqrt{\frac{X_2}{X_1}}. \tag{4}$$

and the boundary conditions a and b are determined in Ref. [1].

It is clearly seen from Eq. (4) that it is possible to obtain the similarity solution for different values of $X_1 = 1 - X_2$. For this purpose, we take values of X_1 within the interval (0, 1) and the following values of parameters $Sc_1 = 1$, $Sc_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$.

The solution of Eq. (3) will be obtained at new boundary conditions for φ_i $(i = 1, 2)$:

$$\varphi_1(0) = \alpha, \quad \varphi_1'(0) = \beta,$$

$$\varphi_2(0) = 1 - \alpha, \quad \varphi_2'(0) = \bar{\theta}_3 \beta, \tag{5}$$

where α and β are determined for different values of $X_1 = 1 - X_2$ so that the conditions $\varphi_i(\infty) = 0$ $(i = 1, 2)$ are fulfilled.X

4. Numerical results

In the case of high soluble gases $(\chi \rightarrow 0)$, it can be obtained directly that $\theta_3 = 0$, and Eq. (3) takes the following form:

$$2f_1''' + f_1 f_1'' = 0, \quad 2\varphi_1'' + Sc_1 f_1 \varphi_1' = 0,$$

$$f_1(0) = 0, \quad f_1'(0) = a, \quad f_1''(0) = b,$$

$$\varphi_1(0) = 1, \quad \varphi_1'(0) = \beta, \tag{6}$$

where β is determined for different values of X_1 so that $\varphi_1(\eta_1^\infty) = 0$ for $\eta_1^\infty = 6$.

The solution of Eq. (6) is obtained for $Sc_1 = 1$, $\theta_1 = 0.1$, $\theta_2 = 0.152$, and the values for dimensionless diffusion flow $\varphi_1'(0)$ are shown in Table 1.

In the case of low soluble gases ($\chi \to \infty$), it can be obtained directly that $\theta_3^{-1} = 0$ and $\varphi_1 \equiv 0$, and Eq. (3) takes the following form:

$$2f_2''' + f_2 f_2'' = 0, \quad 2\varphi_2'' + Sc_2 f_2 \varphi_2' = 0,$$

$$f_2(0) = 0, \quad f_2'(0) = -\frac{a}{\theta_1}, \quad f_2''(0) = \bar{\theta}_2 b,$$

$$\varphi_2(0) = 1, \quad \varphi_2'(0) = \beta, \tag{7}$$

where β is determined for different values of X_2 so that $\varphi_2(\eta_2^\infty) = 0$ for $\eta_2^\infty = 1$. The obtained results for dimensionless diffusion flow $\varphi_2'(0)$ are shown in Table 1.

In the case of gases with middle solubility, the diffusion resistances are commensurable ($\theta_3 \sim 1$) and Eq. (3) should be solved after the introduction of boundary conditions (5) for different values of $X_1 = 1 - X_2$. For this purpose, α and β are selected so that the conditions $\varphi_1(6) = 0$, $\varphi_2(1) = 0$ are fulfilled. The obtained results are shown in Table 2.

Table 1
Numerical results of the boundary conditions (high and low soluble gases)

$Sc_1 = 1$, $Sc_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$					
$\theta_3 = 0$			$\theta_3 \to \infty$		
x_1	$\varphi_1(6)$	$\varphi_1'(0)$	x_2	$\varphi_2(1)$	$\varphi_2'(0)$
0	0.001137	−0.33200	0	0.0022083	−2.5170
0.05	0.000433	−0.31400	0.05	0.0020382	−2.4944
0.1	0.000797	−0.30790	0.1	0.0015402	−2.4850
0.15	0.000747	−0.30546	0.15	0.0019735	−2.4750
0.167	0.000738	−0.30490	0.167	0.0020278	−2.4720
0.2	0.000803	−0.30400	0.2	0.0021716	−2.4660
0.25	0.001137	−0.30290	0.25	0.0017865	−2.4590
0.3	0.000984	−0.30220	0.3	0.0020714	−2.4500
0.35	0.001009	−0.30160	0.35	0.0019494	−2.4420
0.4	0.000974	−0.30112	0.4	0.0019611	−2.4330
0.45	0.000949	−0.30070	0.45	0.0021452	−2.4230
0.5	0.000872	−0.30035	0.5	0.0019807	−2.4130
0.55	0.000933	−0.30000	0.55	0.0025043	−2.4000
0.6	0.000910	−0.29970	0.6	0.0026848	−2.3860
0.65	0.000948	−0.29940	0.65	0.0026329	−2.3705
0.7	0.000884	−0.29915	0.7	0.0028182	−2.3510
0.75	0.000986	−0.29885	0.75	0.0022813	−2.3280
0.8	0.000960	−0.29860	0.8	0.0025298	−2.2945
0.833	0.000940	−0.29842	0.833	0.0028201	−2.2644
0.85	0.000925	−0.29833	0.85	0.0029749	−2.2450
0.9	0.000981	−0.29802	0.9	0.0041870	−2.1572
0.95	0.000974	−0.29767	0.95	0.0093192	−1.9245

It can be seen that diffusion resistances are commensurable in one area along the length of the phase boundary, when

$$\frac{X_1}{X_2} \approx 1. \tag{8}$$

The diffusion resistance in the gaseous (liquid) phase is dominant when $X_1/X_2 \ll 1$ ($X_1/X_2 \gg 1$).

5. Mass transfer kinetics

The mass transfer velocity rate may be expressed [2] by the mass transfer coefficient and the average diffusion flux along the length of the phase boundary.

$$J = k_i \chi^{1-i}\left(c_1^\infty - \chi c_2^\infty\right) = \frac{D_i}{l}\int_0^l \left(\frac{\partial c_i}{\partial y}\right)_{y=0} \mathrm{d}x, \tag{9}$$

$$i = 1, 2,$$

which allows us to further determine the Sherwood number

$$Sh_i = \frac{k_i l}{D_i} = \frac{\chi^{i-1}}{c_1^\infty - \chi c_2^\infty}\int_0^l \left(\frac{\partial c_i}{\partial y}\right)_{y=0} \mathrm{d}x, \quad i = 1, 2. \tag{10}$$

Substitution of Eq. (2) into Eq. (10) leads to:

$$Sh_i = -\sqrt{Re_i}\int_0^1 \frac{\varphi_i'(0)}{\sqrt{X_i}}\,\mathrm{d}X_i, \quad Re_i = \frac{u_i^\infty l}{\upsilon_i}, \tag{11}$$

$$i = 1, 2.$$

6. Comparison between counter-current and co-current flows

The obtained solutions for Eqs. (3), (6) and (7) give the possibility for further determination of mass transfer velocity using the average diffusion flux:

$$J_i = -\int_0^1 \frac{\varphi_i'(0)}{\sqrt{X_i}}\,\mathrm{d}X_i, \quad i = 1, 2. \tag{12}$$

The average diffusion flux values for high ($\theta_3 = 0$), low ($\theta_3 \to \infty$) and middle soluble ($\theta_3 = 1$) gases are shown in Table 3.

For the purpose of comparison between the counter-current flow mass transfer rate and the co-current one, Eq. (3) should be solved [4] using parameters' values corresponding to co-current flow.

Table 2
Numerical results of the boundary conditions (gases with middle solubility)

$Sc_1 = 1$, $Sc_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$							
$\theta_3 = 1$							
x_1	$\varphi_1(0)$	$\varphi_1(6)$	$\varphi_1'(0)$	x_2	$\varphi_2(0)$	$\varphi_2(1)$	$\varphi_2'(0)$
0.9	0.960	0.0012748	−0.2860	0.1	0.040	0.0016793	−0.095333
0.8	0.941	0.0011802	−0.2809	0.2	0.059	0.0021436	−0.140450
0.7	0.925	0.000863	−0.2767	0.3	0.075	0.0011339	−0.181143
0.6	0.908	0.0012521	−0.2720	0.4	0.092	0.0007870	−0.222087
0.5	0.888	0.0014762	−0.2665	0.5	0.112	0.0016322	−0.266500
0.4	0.866	0.0017395	−0.2605	0.6	0.134	0.0004520	−0.319046
0.3	0.836	0.0019449	−0.2523	0.7	0.164	0.0002680	−0.385395
0.2	0.790	0.0011603	−0.2400	0.8	0.210	0.0009160	−0.480000
0.1	0.6995	0.0017781	−0.2150	0.9	0.3005	0.0017972	−0.645001

$$\theta_1^* = -\theta_1 = -0.1, \qquad \theta_2^* = \theta_2 = 0.152,$$

$$f'^*_1(0) = 0.0908, \qquad f''^*_1(0) = 0.37265$$

$$\bar{\theta}_3 = \theta_3, \qquad J_i^* = -2\varphi_i'(0), \quad i = 1, 2. \tag{13}$$

The obtained results for J_i^* $(i = 1, 2)$ are shown in Table 3.

The comparison of these results with values corresponding to co-current flow shows that the counter-current flow mass transfer rate is higher than in case of counter-current flow.

The obtained numerical results in both parts of the work allow us to further determine the ratio between the mass transfer rate and corresponding dissipation energy in case of counter-current and co-current flows:

$$A_i = \frac{Sh_i}{E_i}, \quad A_i^* = \frac{Sh_i^*}{E_i^*}, \quad i = 1, 2.$$

Comparative data for mass transfer energy efficiency (A_i, $i = 1, 2$) in case of counter-current and co-current gas–liquid flows are presented in Table 3. The data show higher efficiency of co-current flow, i.e. higher velocity of mass transfer at equal energy losses.

Table 3
Comparison of the mass transfer energy efficiency

$\theta_3 = 0$		$\theta_3 \to \infty$		$\theta_3 = 1$			
J_1	J_1^*	J_2	J_2^*	J_1	J_1^*	J_2	J_2^*
0.554	0.720	4.380	4.822	0.432	0.626	0.432	0.626
A_1	A_1^*	A_2	A_2^*	A_1	A_1^*	A_2	A_2^*
1.06	1.57	739	750	0.82	1.37	72.8	97.3
E_1	E_1^*	E_2	E_2^*	E_1	E_1^*	E_2	E_2^*
0.525	0.458	0.00593	0.00643	0.525	0.458	0.01328	0.00643

7. Conclusion

The obtained results show that the co-current flow regime is more efficient energetically than the counter-current one because of the lower energy losses at equal rates of the mass transfer. This is a new result in mass transfer theory, where till now the counter-current flow was appreciated because of the possibility to obtain higher average concentration difference [5].

The possibility to change the limiting mass transfer stage along the length of phase boundary presents another new result, which is highly viable in case of middle soluble gases ($\theta_3 \sim 1$).

For low values of X_1 ($X_1/X_2 \ll 1$), the mass transfer is limited by mass transfer in gaseous phase and in the opposite end of phase boundary ($X_1/X_2 \gg 1$) limits the mass transfer in liquid phase.

References

[1] Chr. Boyadjiev, M. Doichinova, Opposite-current flow in gas–liquid boundary layers — Part I. Velocity distribution, International Journal of Heat Mass Transfer 43 (2000).

[2] Chr. Boyadjiev, V. Beschkov, Mass Transfer in Liquid Film Flows, 1st ed., Bulgarian Academy of Sciences Publishing House, Sofia, 1984.

[3] Chr. Boyadjiev, Mass transfer during simultaneous motion of a laminar liquid film and a laminar gas stream, International Journal of Chemical Engineering 11 (1971) 459–464.

[4] Chr. Boyadjiev, Pl. Mitev, On the concentration boundary layer theory at a moving interface, International Journal of Chemical Engineering 14 (1978) 225–228.

[5] V.M. Ramm, Gas Absorption, 1st ed., Chemistry, Moscow, 1976.